

EFFICIENT UNIFORM GRIDS FOR COLLISION HANDLING IN MEDICAL SIMULATORS

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Abstract: We investigate spatial acceleration structures within collision handling in scenarios with "worst-case" spatial layout. These are scenarios where lots of collisions and interactions persist over large time intervals. We focus on acceleration structures based on uniform grids and assess their efficiency in construction, update and query. Z-curves as a technique for the mapping of spatial locality to uniform grids are analyzed to improve the cache-hit rate. Approximate solutions based on the grid representation are considered and discussed in the context of time-critical collision handling. The findings are applied to a deformable collision framework. Experiments are performed on scenarios that are typical for medical simulators. They often exhibit the "worst case" spatial layout mentioned above.

1 INTRODUCTION

Collision handling and the computation of the dynamics are commonly the two main tasks in physically-based animations. While the computation time for the dynamics is generally constant, it can significantly vary for the collision handling dependent on the spatial configuration of the environment. This is due to the fact that spatial acceleration structures are employed. They reduce the search space for collisions to pairs of primitives that are in the same spatial partition. On the other hand, the collision handling can get rather expensive in application scenarios where lots of collisions persist over large time intervals, e. g. the mesh representations of organs in medical simulations are in constant interaction with each other.

Various solutions exist to reduce the computational work in such cases. One is to keep the candidates for intersection tests to a minimum by optimizing the spatial partitioning or by improved culling (Tang et al., 2008). On the other hand, the construction, update and query performance of the spatial acceleration structures could be improved. Time-critical collision handling is a research topic that is concerned with time constraints at the expense of accuracy (Hubbard, 1996).

With respect to dynamics simulations, an approximate collision handling might still yield visually plausible results (O'Sullivan and Dingliana, 2001; Gissler et al., 2009). We focus on the research areas of efficient data structures, parallel algorithms and approximate techniques.

Regarding the spatial partitioning schemes, they can be classified into two main categories, hierarchical approaches and uniform grids. A large body of research has developed hierarchical approaches such as bounding volume hierarchies (BVH) (Cohen et al., 1995; Hubbard, 1996; Klosowski et al., 1998), BSP-trees and kD-trees (Samet and Webber, 1988). Recently, parallel versions of these approaches have been proposed, either for the execution on multi-core CPUs (Lauterbach et al., 2006; Tang et al., 2009), GPUs or both (Kim et al., 2009).

We are concerned with deformable animation and this poses some challenges on BVHs specifically with respect to the construction and update of the hierarchies (Kockara et al., 2007). Therefore, faster refitting approaches have been proposed, e. g., in (Larsson and Akenine-Moeller, 2001). However, in general, BSP trees, octrees or kD-trees are object-dependent. In contrast, spatial subdivision based

on uniform grids is object-independent. Therefore, they are particularly interesting for deformable objects (Teschner et al., 2005).

We investigate algorithms based on uniform grids, in particular spatial hashing (Teschner et al., 2003; Alcantara et al., 2009) and compact grids (Kalojanov and Slusallek, 2009; Lagae and Dutré, 2008). Within this special focus, we analyze the performance of sequential algorithms, how they translate to parallel versions and discuss the special properties of parallelizable data structures. The update of the data structures is of specific interest in the described simulation scenarios with lots of close and persistent proximities and contacts.

Our contribution: We investigate parallel data structures based on uniform grids in terms of efficient construction, update and query. We discuss various representations of uniform grids and their particular properties. We describe Z-curves as a technique for the mapping of spatial locality to uniform grids and analyze their impact on the cache-hit rate. Regarding the index sorting approach, we analyze various sorting algorithms. Approximate solutions regarding the collision query are considered and discussed. The findings are applied in the collision handling step, i. e. in both collision detection and collision response, of a deformable collision framework. Experiments are performed on scenarios from medical simulators. They are specifically challenging because of their spatial layout, i. e., the simulated objects are in constant interaction with each other.

2 RELATED WORK

The problem of collision detection has been extensively studied in the areas of computer graphics, simulation, computational geometry and robotics. For excellent surveys, we refer the reader to (Lin and Manocha, 2003; Ericson, 2004; Teschner et al., 2005; Fares and Hammam, 2005; Kockara et al., 2007). A comprehensive survey on the underlying search methods and data structures can be found in (Bentley and Friedman, 1979). In this section, we focus on the discussion of approaches based on uniform grids.

The projection technique could be considered as a precursor to grids. It sorts a sequence of values according to a key. For example, points in a k -dimensional coordinate frame can be projected onto each coordinate. The k lists that represent the projections can then be obtained by using a standard sorting algorithm k times (Bentley and Friedman, 1979).

(Friedman et al., 1975) used this technique to solve the nearest neighbor problem. Therefore, intervals within the lists that are overlapped by a query rectangle are determined.

Uniform grids discretize the projections into cells and the keys are assigned to the respective cell. (Levinthal, 1966) first applied grids to three-dimensional range queries. Recent research on uniform grids considers the memory requirements and parallelization techniques. (Lagae and Dutré, 2008) propose a compact representation of uniform grids and discuss its application in GPU-based ray tracing. In (Kalojanov and Slusallek, 2009), efficient parallel grid construction is considered in the context of ray tracing. They propose an algorithm for which the performance does not depend on the primitive distribution, because the construction problem is reduced to sorting pairs of primitives and cell indices.

In (Rabin, 1976), hash maps are proposed for a compact representation of a three-dimensional grid. Many hashing techniques have been proposed such as perfect hashing (Fredman et al., 1984), multiple-choice perfect hashing (Pagh and Rodler, 2004) or combinations of both (Alcantara et al., 2009). In (Teschner et al., 2003), an optimized spatial hashing technique for the collision detection of deformable objects is proposed.

Space-filling curves (SFC) feature the ability to preserve spatial locality of an initial domain. In (Griebel and Zumbusch, 1998), SFCs as a method of ordering sparse rectangular grids were introduced. An efficient computation of the Lebesgue space filling curve is proposed in (Pascucci and Frank, 2001). We propose to employ SFCs for the computation of the cell index to increase the efficiency of the discussed collision handling approaches.

3 CONTEXT

We consider the problem of collision handling within a deformable modeling framework and its application to medical simulators. In our framework, objects are represented by tetrahedral meshes. In the collision detection step, we search for point-in-volume collisions to find intersecting objects at discrete time steps, like, e. g., in (Teschner et al., 2003). In the collision response step, forces are computed to resolve the collisions. We employ a penalty-based response scheme that relates the magnitude of the response force of a collided point to its penetration depth. The penetration depth can be estimated consistently for each point using (Heidelberger et al., 2004). This approach requires the computation of intersection points based on

edge-triangle intersection tests. An adjacency structure is used to determine those edges within the mesh that connect a colliding with a non-colliding point.

The point-in-volume test and the edge-triangle intersection test are accelerated by a spatial acceleration structure based on uniform grids. Apart from the primitive tests, the update of the grids is the main computational task in the collision handling. The parallel construction and query of some spatial acceleration structures is not always straightforward, but they are the most crucial steps in the parallelization of the collision handling. Furthermore, as we consider scenes with many close proximities and high spatial and temporal coherence, the acceleration structures should exploit these properties. We discuss the chosen uniform grid structures in more detail in the next section.

4 SPATIAL DATA STRUCTURES

In this section, we describe various spatial data structures based on uniform grids. We briefly summarize the properties of a basic uniform grid. We then specify the extension to a compact grid structure with its benefits in memory requirements and update efficiency. Finally, we discuss spatial hashing to further reduce the memory requirements. All algorithms are designed to allow an efficient parallelization by avoiding race conditions.

4.1 Uniform grid

A uniform grid partitions the simulation domain into regular grid cells of size d . If the domain is bounded by an axis-aligned bounding box (AABB) with \mathbf{e}^{min} and \mathbf{e}^{max} being its minimum and maximum extent, the grid cells can be stored in an array of size $s_x * s_y * s_z$ and $\mathbf{s} = (s_x, s_y, s_z) = \lceil \frac{1}{d}(\mathbf{e}^{max} - \mathbf{e}^{min}) \rceil$. The cell index c of a point with position $\mathbf{p} = (x, y, z)$ can be computed as:

$$c = i + j * s_x + k * s_x * s_y \quad (1)$$

with

$$(i, j, k) = \left(\left\lfloor \frac{x - e_x^{min}}{d} \right\rfloor, \left\lfloor \frac{y - e_y^{min}}{d} \right\rfloor, \left\lfloor \frac{z - e_z^{min}}{d} \right\rfloor \right). \quad (2)$$

Each cell has to store references to all the primitives that overlap the cell. Usually, the references are stored in either linked lists or dynamic arrays. Using dynamic arrays requires more memory, but improves the locality of the references.

4.2 Compact grid

A solution that requires low memory and keeps the locality of the references is the compact grid (Lagae and Dutré, 2008). It consists of two static arrays (see Figure 1). The first is an indirection array L that stores references to primitives. The second array C contains the indexed cells of the grid. Each grid cell stores a pointer to the beginning of an interval within the array L . The end of the interval is implicitly given by the pointer in the adjacent cell within C . Primitives that are referenced within this interval are contained in the respective cell. Thus, the references in L can be seen as sorted according to their cell index.

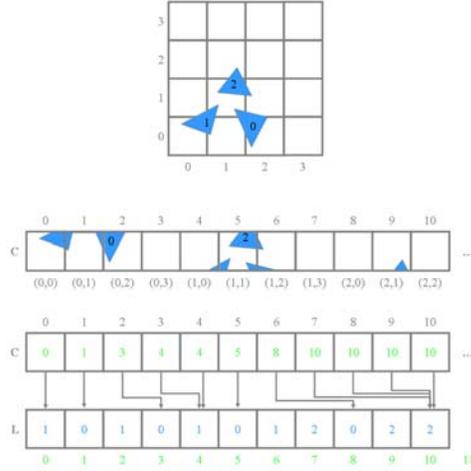


Figure 1: The compact grid representation. The grid cells are lined up in C . Each cell stores an index (green) into L . Together with the index stored in the consecutive cell, an interval is given within L . The interval contains the references (blue) to primitives that intersect this particular cell.

Parallel construction: We employ the algorithm proposed in (Kalojanov and Slusallek, 2009) to construct the compact grid. The advantage of this approach is that it is independent of the primitive distribution, because the construction problem is reduced to sorting. The algorithm first iterates over all primitives and counts how many cells the primitives intersect to reserve space for the array L . The entries of L are computed in a second iteration. An entry consists of a cell index and a pointer to the respective primitive. L is then sorted according to the cell index. Then, all primitives that lie within the same cell are contiguous in L . Parallel reduction is used to compute the offsets that are stored in C in the final step. The complexity and performance of the compact grid construction are defined by the employed sorting algorithm. Details are discussed in

Section 5. All steps can be easily parallelized.

Parallel query: The query is performed by looping in parallel over the corresponding primitives, e. g. the tetrahedrons, per model. The cell index is computed and the primitives, e. g. the points, that are assigned to the same cell are tested for intersection by lookups in L via the offset stored in C .

4.3 Spatial hashing

In contrast to basic uniform grids or compact grids, *spatial hashing* can be employed to subdivide a possibly infinite simulation domain into a regular grid. Therefore, a hash function maps the three-dimensional cells of the infinite grid to an one-dimensional hash-table of finite size (Teschner. et al., 2003). For example, a point with position $\mathbf{p} = (x, y, z)$ is hashed into a hash table of size m by computing its cell index c as follows:

$$c = \left[\left(\left\lfloor \frac{x}{d} \right\rfloor * u \right) \oplus \left(\left\lfloor \frac{y}{d} \right\rfloor * v \right) \oplus \left(\left\lfloor \frac{z}{d} \right\rfloor * w \right) \right] \bmod m, \quad (3)$$

where u, v, w are large prime numbers and d is the cell size. If multiple points are hashed to the same hash cell, chaining is employed to resolve these hash collisions, i. e. the points are stored in a linked list specific to this cell. The construction of such a hash table is difficult to parallelize. It would require serialization of the access to the list structure if two points are hashed to the same cell simultaneously. Furthermore, frequent memory allocations for the linked lists might be necessary if more points move in and out of cells during the simulation. (Teschner. et al., 2003) reserve a certain amount of memory for each list during initialization to avoid this problem, which is quite memory-inefficient. Perfect hashing as proposed in (Fredman et al., 1984) could be employed to avoid hash collision. Thereby, an efficient construction of the hash table would be possible, but at the expense of memory consumption. In the next paragraph, we discuss solutions to these problems.

Cuckoo hashing: A parallel hashing approach is proposed in (Alcantara et al., 2009). It combines the efficiency in construction time of the classical perfect hashing scheme (Fredman et al., 1984) with multiple-choice perfect "cuckoo" hashing (Pagh and Rodler, 2004) that achieves high occupancy. The approach employs a two-level construction. In the first step, the keys are hashed to a set of buckets. The buckets are aligned in one large array B where all keys within the same bucket

are contiguous in memory. Step two works on each bucket independently. The multiple-choice hashing is performed on three hash tables T_0, T_1, T_2 each with its own hash function. Each bucket gets assigned a certain interval within the hash tables. All keys within a bucket are hashed to the first hash table T_0 . If a hash collision occurs, the currently processed key is stored in T_0 and the previously stored key is kicked out. This is repeated iteratively for all keys that are kicked out. In each iteration i , the remaining keys are stored in hash table T_j , with $j = i \bmod d$. It is likely that there is a key which is constantly kicked out of the hash tables. If this is the case, new hash functions have to be chosen and the process has to be repeated entirely. With increasing hash table size, this is becoming unlikely and negligible in practice.

The key value is the cell index. Naturally, as the primitives within the same grid cell get the same cell index, the approach has to be extended to multi-valued hashing. Therefore, each key gets a counter and an index pointer in order to know how many values it represents within the hash table and where to find those values within a secondary buffer array.

Discussion: The data structures of the compact grid and cuckoo hashing are remarkably similar. The secondary buffer array resembles L and the hash tables replace C . In contrast to the compact grid, C does not scale with the simulation domain, but with the number of primitives. The array sizes for the buckets and hash tables are reportedly chosen such that the occupancy reaches 80% for the buckets and 70% for the hash tables on average (Alcantara et al., 2009).

In this section, we have described two approaches based on uniform grids for efficient construction and collision query. In the next section, we discuss implementation details and decisions we make that make the approaches specifically suited for medical simulators where the close proximities and many interactions between the models provide a challenge.

5 IMPLEMENTATION

In this section, we discuss the implementation details and the influence of parameters on the efficiency of both approaches. In general, the reduced memory requirements and the increased efficiency compared to the basic uniform grid can be attributed to the fact that L is a static array (Lagae and Dutré, 2008). On the other hand, the static array demands a reconstruction from scratch in dynamic scenes if the number of references to primitives varies from frame to frame

and, thus, changes the size of L . In the following we discuss three aspects related to these arguments.

Cell size: The cell size influences the number of primitive pairs that have to be tested for intersection. For large cells, the number of primitives per cell increases. For small cells, the bounding boxes of the primitives might cover a large number of cells. In (Teschner. et al., 2003), it is suggested that the average edge length of all tetrahedrons should be chosen to achieve optimal performance. For the algorithms with sorting, the second criterion has more severe implications. The more cells the primitives cover, the larger the data array gets and the longer the sorting takes. On the other hand, the number of non-culled primitive pairs increases with the cell size. Therefore, in general, we stick to the recommendation and use the average edge length. However, if the tetrahedrons are close to regular, the maximal edge length is chosen.

Parallel sorting: The complexity and performance of the compact grid construction are dominated by the employed sorting algorithm. We tested a parallel radix sort and a parallel re-implementation of the sorting algorithm of the Standard Template Library (STL) of C++. The last one is part of the OpenMP Multi-Threaded Template Library (Beekhof, ; Ope, 2005). Inherently, the radix sort does not take advantage of pre-sorted sets of keys. Thus, its performance is constant. On the other hand, the STL-sort benefits from sets of keys that are predominantly sorted. Such sets appear if the spatial configuration of objects in a simulation domain is temporally coherent i. e. is similar to the previous frame. In such a case, only a small number of keys moves to new spatial cells. A sorted set is quickly re-established. However, using STL-sort on a largely distorted set of keys might prove to be slower than radix sort depending on the input size. In such cases it might be beneficial to employ the radix sort algorithm. Therefore, the amount of distorted keys is determined by keeping track of the primitives' bounding boxes. If too many bounding boxes move into new spatial cells, a threshold triggers the switch to radix sort, and back. We discuss timings for both sorting algorithms in the results section.

Z-curves: Primitives that intersect more than one cell have to query the primitives stored in all the intersected cells. It depends on the indexing function, whether the order of the referenced primitives in L is memory-coherent, i. e. are likely to be contiguous in memory. Space-filling curves provide a so-

lution to this problem. They are a common tool in computer science for mapping multidimensional data to one dimension while preserving spatial locality as good as possible. We propose to replace equation (1) which orders the cells according to their Z-value by the Lebesgue space filling curve, also called Z-order space filling curve, to construct an array C that is more spatially compact (see Figure 2). We employ the Z-curves, because they can be efficiently computed by bit-interleaving (Pascucci and Frank, 2001). Details on the performance are discussed in Section 6.

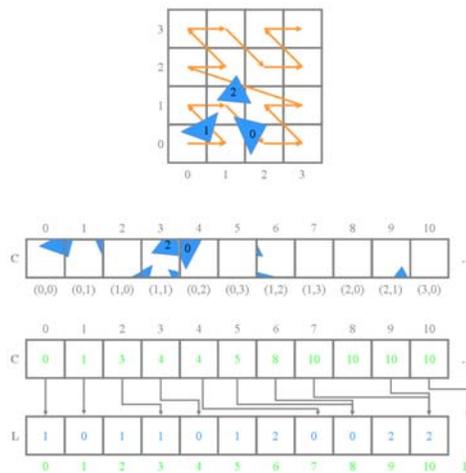


Figure 2: The Z-curve changes the line up of the grid cells in C to preserve the spatial locality. The intervals in L are re-arranged, too.

Approximate collision detection: If the number of references to primitives varies, L has to be resized and constructed from scratch. In this case, the spatial locality established by the index sort cannot be exploited in the sorting process anymore and the computation time for the sorting of L is increased. This computational overhead can be avoided, if we assume a maximal number of cell overlaps per primitive bounding box and allocate sufficient memory for array L during the initialization of the simulation. Thus, we avoid the rebuild from scratch. However, the array size should not deviate too much from the size actually needed, since it has to be sorted. We make the assumption that eight cells suffice to cover the primitives' bounding boxes if the cell size is chosen to be the maximal edge length. We experienced less additional sorting time than what is needed if spatial locality has to be re-established over the whole array L . The assumption might not hold if the primitive is stretched through deformation. In this case, we approximate the collision handling. Therefore,

	stacking	eye	skull
scene statistics			
#points	4840	3167	8293
#edges	19620	16248	38352
#tris	9600	6472	13592
#tetas	10000	9819	23225

Table 1: Scene statistics for the three test scenarios.

we share the available cells among the cells actually covered by the primitives’ bounding box by random assignment. Thus, cells not represented in L might involve collision information that is missed. However, we observed no visually implausible behavior in the simulation results. This is in agreement with research results on collisions and perception (O’Sullivan and Dingliana, 2001). If accurate collision handling is required, L is resized and built up from scratch.

6 RESULTS

In this section, we evaluate the performance of the presented methods in the context of interactive deformable modeling. We compare to traditional grid representations and analyze the construction and query times of the approaches using a set of test scenarios. Further, we discuss the impact of the cell size on the performance and how the parallel approaches scale with the number of threads.

Setup: We have integrated the approach into a deformable modeling framework based on the Finite Element Method for tetrahedrons (Müller and Gross, 2004) to accelerate the collision handling. The timings have been obtained on a commodity computer with one quad-core 2.66 GHz Intel Xeon E5430 CPU, 12 MB L2 cache and 4 GB of memory. For the scaling experiments, a second computer with two quad-core 3.16 GHz Intel Xeon X5460 CPUs, 2x6 MB L2 cache and 16 GB of memory has been used. The number of cores is given with the timings, respectively. The methods are implemented in high-level C++ with STL. No low-level optimization such as SIMD is used. Parallelization of the code is achieved using OpenMP (Ope, 2005).

Test scenarios: The framework is applied to three test scenarios. Their statistics are given in Table 1. Stacking of deformable membranes is performed in the first scene (see Figure 3) Here, the number of collisions increases until all membranes are stacked up.

In the second and third scene, we apply the framework



Figure 3: Objects are stacked up. In the equilibrium, many contacts persist over large time intervals. The index sorting experiences a huge efficiency gain the more interactions occur.

to medical simulations. In the eye data set (see Figure 4), the interaction between skull, fat tissue, skin tissue, eye muscle, eye nerve, eye bulb and titanium mesh is simulated. The titanium mesh is used in orbital reconstruction to fix fractures to the orbital floor (see Figure 8). Thus, the eye bulb is repositioned. All objects are in constant interaction. In the skull data set, the interaction of the soft tissue with the skull, upper jaw and lower jaw is simulated. The lower jaw is repositioned and the effect on the skin tissue is simulated (see Figure 5).

Index sort vs. serial hashing: First, we compare the parallel index-sort approach (IS) to the serial hashing approach (SH) (Teschner. et al., 2003). We observe a more efficient update of the points stored in the static array of IS when compared to the repeated insertion of the points into the dynamic arrays within the hash cells, even when using only one core. The query is slower in IS when using one core, due to the standard parallelization technique of adding one additional iteration to determine the size of the output array in order to write out the collisions in parallel. However, this is quickly compensated with each additional core, see Figure 6. The first frame shows a high initial computation time, since the array L is sorted for the first time and spatial locality is established in L . The frame rate stays interactive with the compact grid approach, even when all membranes are stacked up.

Index sort vs. Cuckoo hashing: We set the size of the buckets and the size of the hash table so that an occupancy of 71% is achieved on average. Overfilling of the buckets or hash collisions that enforces a repeated insertion has never occurred in our test runs, so we refer to the empirical results in (Alcantara et al., 2009). The index sort approach again shows superior insertion times with respect to points, but falls back when inserting the footprints of a large number of colliding edges. On the other hand, the cuckoo hashing introduces some overhead. This is due to the multiple hash key computations for

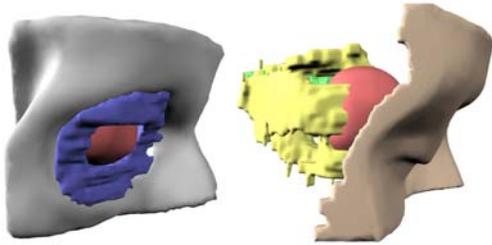


Figure 4: The orbital reconstruction data set. Left: eye (red), fat tissue (blue) and skull (gray). Right: face tissue (brown), eye (red), muscle (yellow) and nerve (green).

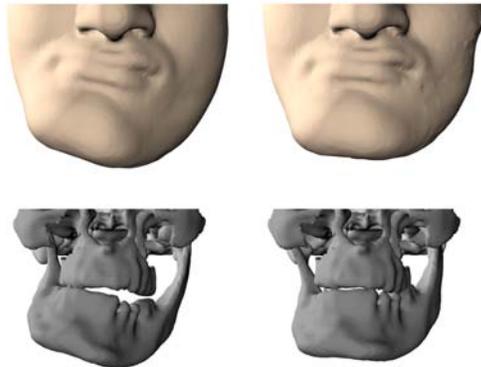


Figure 5: the prediction of skin-tissue deformations due to bone realignments supports the preoperative planning in craniomaxillofacial surgery. The lower jaw is repositioned and the effects on the skin tissue is computed.

the three hash tables and keys that iteratively have to find an empty hash cell. Performances are given in Figure 7. The parallel scaling is discussed in the following.

Parallel scaling: Ideally, the performance gain from parallel algorithms should be linear in the number of cores. However, this cannot be expected for several reasons. First, there is some parallelization overhead for synchronization and communication between different threads. Second, some parallel algorithms have to perform additional computations that only pay off after providing a certain amount of additional cores, e. g. additional loops over data arrays. Third, certain portions of an algorithm cannot be parallelized. According to Amdahl’s law, this limits the achievable speedup (Amdahl, 1967). For example, if 90 percent of the algorithm can be parallelized, the maximum speedup is 10, regardless of the number of cores. Note that the law assumes that the problem size remains the same when parallelized.

The proportion of a program that is run in

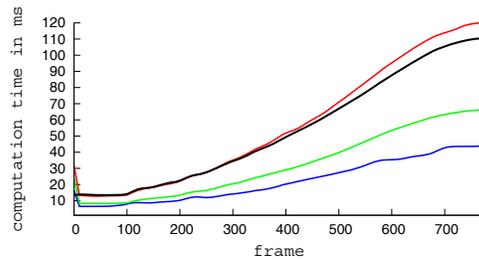


Figure 6: Collision-handling timings for the stacking scene using spatial hashing (black) and index sort with one (red), two (green) and four (blue) cores.

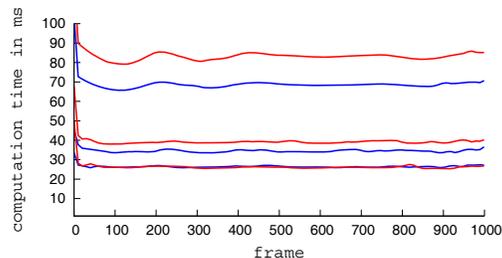


Figure 7: Timings for the eye scene using index sort (red) and cuckoo hashing (blue) with one, four and eight cores.

parallel can be estimated using: $P_{estimated} = \left[\frac{1}{S_{measured}} - 1 \right] / \left[\frac{1}{\#processors} - 1 \right]$.

We measure the speedup using the dual quad-core machine with all eight cores and get an average speedup of 3.2 for the index sort approach and 2.6 for the cuckoo hashing (based on the data shown in Figure 7). Using the above equation, the estimated amount of code is 80% for the index sort approach and 70% for the cuckoo hashing approach. We see two reasons for these results. Regarding the index sort approach, the STL-sort shows poor scaling behavior which leads to a speedup of 1.5 for the update functions. This is compensated by speedups of 5 in the query functions. Regarding the cuckoo hashing, the work is distributed to the threads per bucket. When multi-value hashing is performed, the fixed size of the buckets’ hash tables leads to unequal work loads in the threads. Dynamically sized tables would account for this, but would require additional hashing to determine the actual number of values hidden behind the unique keys.

Sorting performance: As is discussed in Section 5, two parallel sort algorithms have been implemented. The results support the assumptions made. For an input size of 128000 entries in L , the radix sort takes 14ms on the 4-core system. The STL-sort takes less than 6ms if 10% of the keys change their value and

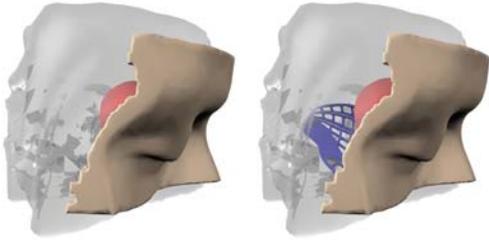


Figure 8: Deformable modeling supports the preoperative planning in craniomaxillofacial surgery. In orbital reconstruction, a titanium mesh is placed beneath the bulb for repositioning. All simulated objects are in constant interaction. The spatial configuration only changes slowly over time.

3ms for 2%. A randomly filled array performs about equally in both approaches. Thus, the STL-sort is always to be preferred for scenes with a complexity like the ones we show here.

Z-curve reordering: We apply the Z-curve to Equation 1 to increase the spatial locality in memory. The index sort only profits marginally when rearranging L . However, the query of edge-triangle intersections gets a performance boost by about 8% on average. When querying the intersection for one edge, the triangles that are spatially close and likely to intersect are also close in memory and likely to be already loaded into the cache.

Approximate collision detection:

In Section 5, we have proposed a simple acceleration technique based on the index sort approach. Therefore, we set L to be a static array in the next experiment. Further, we set for each primitive a fixed number of cell entries it can write to, i. e. 27, 8 and 1 in consecutive test runs. Its impact on performance as well as on the introduced average error in the simulation are given in Table 2. The error is measured as the Euclidean distance a deformed point is deviating from the original simulation result, i. e. the result achieved using a dynamic array L that provides the number of cells actually needed by the primitives. We take the final simulation result of the skull scene, i. e. the simulation has reached an equilibrium. We set the cell size to be the average edge length. Errors below one millimeter can be tolerated in this field of applications. We observe a performance boost of factor 2.8 when switching from 27 to 1 cell per primitive in the eye scene and of factor 1.9 when switching from 27 to 8 cells in the skull scene.

	eye			skull		
	27	8	1	27	8	1
#cells per primitive	27	8	1	27	8	1
max. deviation [cm]	1.0*	0.03	0.034	3.0*	0.12	0.22
avg. deviation [cm]	1.3*	0.002	0.024	1.6*	0.0008	0.003
standard deviation [cm]	1.2*	0.003	0.057	9.1*	0.0045	0.009

Table 2: Approximate collision handling: timings for the medical scenes with a fixed numbers of cells in L and the measured errors in cm. Errors within 1 millimeter deviation are tolerable.

7 CONCLUSION

We have presented two acceleration data structures based on uniform grids for the efficient collision handling in a deformable modeling framework. Important aspects critical to the performance of such a system were discussed. We have analyzed Z-curves for the mapping of spatial locality to the grid representations. Further, the STL-sorting algorithm exhibits better performance than the radix sort when applied in the index sort approach. However, improved parallel sorting algorithms have to be developed to achieve better speedups. Finally, we have shown how the collision handling can be approximated using the spatial data structures for an additional increase in performance. We have analyzed the performance aspects of the presented uniform grid approaches and gave a detailed scaling analysis.

The efficient update of the data structures as well as the efficient query specifically improve the performance in medical simulation scenarios where lots of collisions persist over large time intervals. In future work, we would like to investigate improved culling schemes such as (Tang et al., 2008) and incorporate the findings into our framework.

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